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Math 102-Review Problems
(55 Problems)

1. Write the sum $\frac{4}{3}+\frac{9}{4}+\frac{16}{5}+\frac{25}{6}+\frac{36}{7}+\frac{49}{8}+\frac{64}{9}$ in sigma notation.
2. Find the value of the sum $\sum_{i=12}^{50} i^3$. **Answer:** 1621269.
3. Find the sum $\sum_{i=1}^{n}\left(2+\frac{i}{2n}\right)^2$. **Answer:** $\frac{122n^2+27n+1}{24n}$.
4. Find the limit $\lim_{n\to+\infty}\sum_{k=1}^{n}\left(\frac{k}{5n^2}-\frac{2}{n}\right)$. **Answer:** $\frac{-19}{10}$.
5. Find the sum $\sum_{i=1}^{n}\left(\frac{1}{i}-\frac{1}{i+1}\right)$. **Answer:** $\frac{n}{n+1}$.
6. Use the midpoint rule with $n=3$ to estimate the definite integral $\int_1^4 \frac{1}{x^2}dx$.
7. Find $\int(\cos x+\sin x)^2 dx$. **Answer**: $x-\frac{\cos(2x)}{2}+C$.
8. Find $\int\frac{e^{3x}+e^{-3x}}{e^x+e^{-x}}dx$. **Answer:** $\frac{1}{2}e^{2x}-\frac{1}{2}e^{-2x}-x+C$.
9. Find $\int\frac{\sin x+\tan x}{1+\cos x}dx$. **Answer**: $\ln|\sec x|+C$.
10. Find $\int\cos(f(t))f'(t)dt$. **Answer:** $\sin(f(t))+C$
11. Find $\lim_{x\to 0}\frac{\int_0^x(1-e^{-t^2})dt}{\cos x-1}$. **Answer:** 0.
12. Find $\int\frac{1}{e^{-x}\sqrt{e^x+7}}dx$. **Answer:** $2\sqrt{e^x+7}+C$.
13. Find $\int x\sqrt{x+1}\ dx$. **Hint:** let $u=x+1$.
14. Evaluate $\int_{-1}^{1}[\tan(x^3)+2\sqrt{1-x^2}]dx$. **Answer:** π.
15. Find the area of the region lying between the curve $y=x^2-x$ and the $x-axis$ over the interval [0, 2]. **Answer:** 1.
16. Find the area of the region enclosed by the curves $y=\ln x, y=1, x=1$. **Answer:** $e-2$.
17. Let R be the region enclosed by the curves $y=x^2, y=0, x=2$. Find the volume of the solid generated by rotating R about
    a. the $x-axis$. **Answer:** Disk Method, $\pi\int_0^2 (x^2)^2 dx=(32/5)\pi$.

b. the line $y=-2$. **Answer:** Washer Method, $\pi\int_0^2\left[(x^2-(-2))^2-(0-(-2))^2\right]dx$

c. the line $y=5$. **Answer:** Washer Method, $\pi\int_0^2\left[(5-0)^2-(5-x^2)^2\right]dx$.

d. the $y-axis$. **Answer:** Washer Method, $\pi\int_0^4\left[2^2-(\sqrt{y})^2\right]dy=8\pi$.

e. the line $x=-1$. **Answer:** Washer Method, $\pi\int_0^4[(2-(-1))^2-(\sqrt{y}-(-1))^2]dy$

f. the line $x=2$. **Answer:** Disk Method, $\pi\int_0^4(2-\sqrt{y})^2\,dy$.

g. the line $x=3$. **Answer:** Washer Method, $\pi\int_0^4[(3-\sqrt{y})^2-(3-2)^2]dy$.

18. Find the volume of the solid whose base is the region enclosed by the curves $y=x$ and $y=x^2$, and whose cross sections perpendicular to the $x-axis$ are equilateral triangles. **Answer:** $\sqrt{3}/120$.
19. Find the volume of the solid whose base is enclosed by the curves $y=x^2, y=0, x=2,$ and whose cross sections perpendicular to the $y-axis$ are squares. **Answer:** 8/3.
20. Let R be the region lying between the curve $y=3x\sin(x^3)$ and the $x-axis$ over the interval $[0,\sqrt[3]{\pi}]$. Find the volume of the solid generated by rotating R about the $y-axis$. **Answer:** 4π.
21. Evaluate $\int_{\pi/4}^{\pi/2}\cos x\ln(\sin x)dx$. **Answer:** $-1-\frac{\sqrt{2}}{2}[\ln(\frac{\sqrt{2}}{2})-1]$.
22. Find $\int(e^{\sin t}\sin t\cos t)dt$. **Answer:** $e^{\sin t}(\sin t-1)+C$.
23. Find $\int(e^t\sin t\cos t)dt$.
24. Find $\int\sqrt{x}\sin(\sqrt{x})dx$. **Answer:** $-2x\cos(\sqrt{x})+4\sqrt{x}\sin(\sqrt{x})+4\cos(\sqrt{x})+C$.
25. Find $\int\csc^3(2x)dx$.
26. Find $\int\frac{1-\sin^4 t}{1-\sin t}dt$.
27. Find $\int\sqrt{a^2-x^2}dx,(a>0)$. **Answer:** $\frac{1}{2}[x\sqrt{a^2-x^2}+a^2\sin^{-1}\left(\frac{x}{a}\right)]+C.$
28. Find $\int e^x\sqrt{3-e^{2x}}\ dx$. **Answer:** $\frac{1}{2}[e^x\sqrt{3-e^{2x}}+3\sin^{-1}\left(\frac{e^x}{\sqrt{3}}\right)]+C.$
29. Find the area of the part of the circle $x^2+y^2=2$ that lies in the first quadrant over the interval [0, 1]. **Answer:** $(2+\pi)/4$.

30. Decompose $\frac{4x-5}{x^3-3x^2}$ into partial fractions. **Answer:** $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}$, $A=\frac{-7}{9}, B=\frac{5}{3}, C=\frac{7}{9}$.

31. Find $\int \frac{e^{4t}}{e^{2t}-2}dt$. **Answer:** $\frac{1}{2}e^{2t}+\ln\left|e^{2t}-2\right|+C$.

32. Find $\int \frac{x}{x^2+2x+5}dx$. **Answer:** $\frac{1}{2}[\ln(x^2+2x+5)-\tan^{-1}(\frac{x+1}{2})]+C$.

33. Find $\int \frac{1}{(x^2-1)^2}dx$.

34. Find $\int \frac{1}{2x^3+x^2+2x+1}dx$. **Answer:** $\frac{1}{5}[2\ln|2x+1|-\ln(x^2+1)+\tan^{-1}x]+C$.

35. Find $\int \frac{1}{\sin x+\tan x}dx$. **Answer:** $\frac{1}{4}[2\ln\left|\tan(\frac{x}{2})\right|-\tan^2(\frac{x}{2})]+C$.

36. Determine whether the integral $\int_0^{+\infty}\frac{1}{x^2}dx$ converges or diverges. **Answer:** Diverges.

37. Consider the curve $y=x^3+\frac{1}{12x}$ defined over the interval [1, 7].
    a. Find the arc length function of the given curve. **Answer:** $s(x)=x^3-\frac{1}{12x}-\frac{11}{12}$.
    b. Find $s(1)$ and $s(3)$. Interpret your answers.

38. Find the limit of the following sequences $\{a_n\}_{n=1}^{+\infty}$, where:
    a. $a_n=\frac{1}{n!}$. **Answer:** 0.
    b. $a_n=\frac{(-1)^n\sin(3n)}{n}$. **Answer:** 0.
    c. $a_n=\frac{(n+1)^n}{n^n}$. **Answer:** e.

39. Determine whether the sequence $\{n+(-1)^n\}_{n=1}^{+\infty}$ is increasing, decreasing, or neither.

40. Show that the sequence $\{\frac{2n}{n+1}\}_{n=1}^{+\infty}$ is bounded.

41. TRUE or FALSE:
    a. If the series $\sum_{n=1}^{+\infty}a_n$ converges, then $\lim_{n\to+\infty}a_n=0$.
    b. If $\sum_{n=1}^{+\infty}a_n$ converges and $\sum_{n=1}^{+\infty}b_n$ diverges, then $\sum_{n=1}^{+\infty}(a_n+b_n)$ diverges.
    c. If $\sum_{n=1}^{+\infty}a_n$ diverges, then $\sum_{n=1}^{+\infty}5a_n$ diverges.

42. Determine whether the series $\sum_{n=0}^{+\infty}\frac{(-2)^{3n+1}}{5^{2+2n}}$ converges or diverges. If it converges, find its sum. **Answer:** The series converges and its sum is -2/33.
43. Express the number 0.2131313… as a ratio of two integers. **Answer:** 211/990.
44. Determine the convergence or divergence of each of the following series.
    a. $\sum_{n=1}^{+\infty} n\sin(\frac{1}{n})$. **Answer:** D.
    b. $\sum_{n=1}^{+\infty}\frac{1}{\sqrt[n]{n}}$. **Answer:** D.
    c. $\sum_{n=1}^{+\infty}\frac{1}{\sqrt[5]{n^3+3n^2+6n+6}}$. **Answer:** D.
45. Determine whether the series $\sum_{n=1}^{+\infty}\left(\frac{\ln n}{n}\right)^3$ converges or diverges. **Answer:** C.
46. Classify each of the following series as absolutely convergent (AC), conditionally convergent (CC), or divergent (D).
    a. $\sum_{n=1}^{+\infty}(-1)^n\frac{\sqrt[3]{n+1}}{4n+5}$.. **Answer:** CC.
    b. $\sum_{n=1}^{+\infty}(-1)^{n-1}\frac{(n+1)^n}{(n+1)!}$. **Answer:** D.
    c. $\sum_{n=1}^{+\infty}(-1)^n\frac{2\cdot 4\cdot 6\cdots(2n)}{[(2n-1)!]^2}$. **Answer:** AC.
47. Determine whether the series $\sum_{n=1}^{+\infty}(-1)^{n-1}(\sqrt[n]{2}-1)$ converges or diverges.
48. Determine whether the series $\sum_{n=1}^{+\infty}\frac{(-1)^{[\sqrt{n}]}}{n^5}$ converges or diverges. Here $[\sqrt{n}]$ denotes the greatest integer less than or equal to $\sqrt{n}$.
49. Determine whether each of the following series converges or diverges.
    a. $\sum_{n=1}^{+\infty}\frac{e^n}{(2n+1)!}$. **Answer:** C.
    b. $\sum_{n=2}^{+\infty}\frac{1}{n(\ln n)^2}$. **Answer:** C.
    c. $\sum_{n=1}^{+\infty}\left(\frac{n-1}{n}\right)^{n^2}$. **Answer:** C.
50. Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{+\infty}\frac{(2x-1)^2}{3^{n+1}\ln n}$.
51. Find a power series representation for $f(x)=\ln(x+1)$. Find the interval of convergence.
52. Starting from the origin, a particle moves 1 m to the right, and then 1/2 m to the left, and then 1/3 m to the right, and then 1/4 m to the left, and so on indefinitely. How far from the origin will the particle be.
53. Find the first three terms of the Taylor series of $f(x)=x\sin x$ at $x=\pi/2$.

54. Find the sum of the series $\sum_{n=1}^{+\infty} \frac{(-1)^n 2^n}{n!}$. **Answer:** $\frac{1-e^2}{e^2}$.

55. Simplify the expression $\frac{(n+1)!}{3 \cdot 6 \cdot 9 \cdots (3n)}$.

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